Today: PP and binomial, Gamma, exponential RV’s

Next: PK 5.4

This week:

- Homework 7 (due Friday, March 13, 11:59 pm)
- Regrades Midterm 2 Tuesday, March 10, 8 am - 11 pm
- Regrades HW5 Tuesday, March 10, 8 am - 11 pm
Poisson Process and Binominal distribution

Thm. Let \((X_t)_{t \geq 0}\) be PP of rate \(\lambda > 0\). Then for \(0 < u < t\) and \(0 \leq k \leq n\)

\[
P(X_u = k \mid X_t = n) = 
\]

Proof. \[
P(X_u = k \mid X_t = n) = 
\]
Example (PK, Exercise 5.3.3)

Customers enter a store according to a Poisson process of rate $\lambda = 6$ per hour. Suppose it is known that only a single customer entered during the first hour. What is the conditional probability that this person entered during the first 15 minutes?
Waiting times and sojourn times

$W_i$ is called arrival time / occurrence time / $i^{th}$ event waiting time

$S_i$ is called sojourn time / interarrival time
**Distribution of Wi’s**

**Thm.** Let $(X_t)_{t \geq 0}$ be a PP with rate $\lambda > 0$. Then for $n \geq 1$

**Proof.** Compute the c.d.f. $F_{W_n}(t) = P(W_n \leq t)$

Compute p.d.f. from c.d.f. (differentiate)
Joint distribution and joint density

Definition. (Joint distribution). Let \( X = (X_1, X_2, \ldots, X_n) \) be a random vector. Then the joint distribution function of \( X \) is defined by

\[
F_X(x_1, x_2, \ldots, x_n) =
\]

If \( X \) has a joint density \( f_X(x_1, x_2, \ldots, x_n) \), then

\[
F_X(x_1, x_2, \ldots, x_n) =
\]

In particular

and

\[
(*) \quad \int_{x_1}^{x_1+\Delta x_1} \int_{x_2}^{x_2+\Delta x_2} \ldots \int_{x_n}^{x_n+\Delta x_n} f_X(u_1, \ldots, u_n) \, du_1 \cdots du_n =
\]
Thm (Thm 5.5). The sojourn times $S_0, S_1, \ldots, S_{n-1}$ are i.i.d. random variables, $S_0 \sim \text{Exp}(\lambda)$.

Proof. Let $f_S(s_0, s_1, \ldots, s_{n-1})$ be the joint density of $S = (S_0, \ldots, S_{n-1})$. Enough to show that $f_S(s_0, s_1, \ldots, s_{n-1}) = (\lambda e^{-\lambda s_0})(\lambda e^{-\lambda s_1}) \cdots (\lambda e^{-\lambda s_{n-1}})$.

(a) Compute $\int_{s_0}^{s_0+\Delta s_0} \int_{s_1}^{s_1+\Delta s_1} f_S(u_0, u_1) du_0 du_1$.

(b)
Proof. Upper bound

(i) Upper bound

\[ P \left( S_0 \in (s_0, s_0 + \Delta s_0], S_1 \in (s_1, s_1 + \Delta s_1] \right) \]
(ii) Lower bound:

\[ P(S_0 \in (s_0, s_0 + \Delta s_0], S_1 \in (s_1, s_1 + \Delta s_1]) \geq \]