Today: PP and uniform RV's

Next: PK 5.4

This week:

- Homework 7 (due Friday, March 13, 11:59 pm)
- Regrades Midterm 2 Tuesday, March 10, 8 am - 11 pm
- Regrades HW5 Tuesday, March 10, 8 am - 11 pm
- CAPEs
Warm-up computations

Let \((X_t)_{t \geq 0}\) be a PP of rate \(\lambda > 0\). Denote by \(W_i\) the \(i\)-th waiting time.

\[ P( W_i \leq \omega, \mid X_t = 1 ) = ? \]
Thm. (Thm 5.7). Let \((X_t)_{t \geq 0}\) be a PP of rate \(\lambda > 0\).

Let \(W_1, W_2, \ldots\) be the waiting (occurrence) times of \((X_t)_{t \geq 0}\).

Conditioned on the event \(\{X_t = n\}\), the rv's \(W_1, W_2, \ldots, W_n\) have the joint density
Relation with $\text{Unif}[0,t]$ 

Let $U_1, U_2, \ldots, U_n$ be i.i.d. r.v.'s, $U_i \sim \text{Unif}[0,t]$.

Denote by $V_i$, $1 \leq i \leq n$, the r.v. giving the $i$-th smallest number of the set \{ $U_1, U_2, \ldots, U_n$ \}.
Derivation of the density of \((V_1, \ldots, V_n)\)

**Proof for** \(n=2\).

\[
P(\sigma_1 < V_1 \leq \sigma_1 + \Delta \sigma_1, \sigma_2 < V_2 \leq \sigma_2 + \Delta \sigma_2)
\]

**Proof for** \(n>2\).

\[
P(\sigma_1 < V_1 \leq \sigma_1 + \Delta \sigma_1, \sigma_2 < V_2 \leq \sigma_2 + \Delta \sigma_2, \sigma_3 < V_3 \leq \sigma_3 + \Delta \sigma_3, \ldots)
\]
Proof of thm 5.7

Statement:

\[ f_{w_1,w_2,\ldots,w_n | X_t}(w_1,w_2,\ldots,w_n | n) = n! t^{-n} \text{ for } 0 < w_1 < w_2 < \cdots < w_n < t \]

\[ P \left( w_i < W_i \leq w_i + \Delta w_i, w_2 < W_2 \leq w_2 + \Delta w_2, \ldots, w_n < W_n \leq w_n + \Delta w_n | X_t = n \right) \]
Example (PK, p 249)

Customers arrive according to PP with rate \( \lambda > 0 \).
Each customer pays 1$ on arrival.
Each dollar paid at time \( s \) has a discounted present value of \( e^{-\rho s} \). What is the discounted value of the expected total sum collected before time \( t \) ?
Example (PK, p. 249)

\[ E \left( \sum_{k=1}^{X_t} e^{-\beta W_k} \right) = \]